

## Left and Right with a Graphing Calculator

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activity addresses the Data Analysis and Probability Standard for grades 9-12 of the National Council of Teachers of Mathematics (NCTM 2000), by helping students learn to select and use appropriate statistical methods to analyze data. Students work with bivariate data, display a scatterplot, describe its shape, and determine lines of best fit using technological tools.

In a mathematics methods course, each one of the future secondary teachers was asked to conduct a session in class in the same way they would conduct it in a high school. Their fellow students in turn participated in the activity as if they were high school students. We will refer to the participants as students, and to the person conducting the activity as teacher.

Students were intrigued when the teacher asked them to write the letters of the alphabet at the beginning of her high school mathematics lesson. “Hold your pen in your right hand. You will write the letters of the alphabet continuously for 30 seconds. If you get to the end of the alphabet, start from the beginning again.” When the teacher gave the indication to start, students wrote the letters. For most of them it was an easy task, but a couple of students struggled to write the letters. After 30 seconds the teacher asked them to stop. Then the teacher asked students to repeat the activity with their left hand. Now it was the majority of the students who struggled to write the letters, and for a couple of students it was a breeze. Students then counted how many letters they had written with each hand and wrote down the numbers. The teacher collected the data for the number of letters written in 30 seconds with each hand from each student in the class, wrote the numbers at the board (see Table 1), and then entered the two lists into the graphing calculator. Table 1 displays the data for the ten participants.

x	number of letters right hand	74	18	52	52	13	32	55	56	54	45
y	number of letters left hand	26	59	23	22	56	75	26	26	30	18

Table 1: Number of letters written with each hand

The teacher adjusted the window in the calculator to display the data as a scatter plot (Figure 1). The  $x$ -values are the number of letters written with the right hand, the  $y$ -values the number of letters written with the left hand. The teacher projected the graph so that every student could see it.

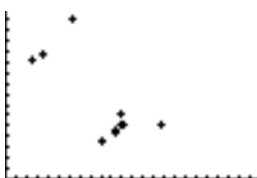


Figure 1: Scatter plot of letters written with each hand

### Analyzing the graph

The teacher led the discussion about the graph. Students drew the following conclusions. They noticed that points seem to cluster in two groups. Adding the graph of the line  $y = x$ , students could see this line clearly separates the two groups (Figure 2). Points above the line represent people who wrote more letters with their left hand than with their right hand, presumably, left-handed people. Points below the line represent right-handed people. Points very close to the line  $y = x$  would represent people who are ambidextrous (Lanius, 1999).

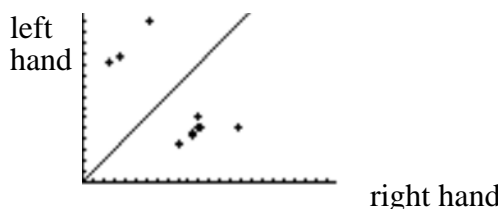


Figure 2:  $y = x$  divides the two groups

You may collect data in your own classroom (or you may use the data presented here). Enter into lists L1 and L2 the number of letters written with the right hand and left hand respectively for the students in your class (here the examples are given for the data from Table 1). The window in the calculator can be set to the following values  $X_{\min} = 0$ ,  $X_{\max} = 119$ ,  $X_{\text{scl}} = 5$ ,  $Y_{\min} = 0$ ,  $Y_{\max} = 79$ ,  $Y_{\text{scl}} = 5$ ,  $X_{\text{res}} = 1$ . With this window, the graph of  $y = x$  will form a  $45^\circ$  angle.

### Line of best fit

There are two methods commonly used to fit a line to a scatterplot. One is the median-median regression method; the other method is finding a line of regression using the least-squares fit. The median-median method is a simple way to introduce the idea of fitting a line to data. It is also a method that is robust to outliers (Walters, Morrell, & Auer, 2006). The least-squares line minimizes the sum of the squares of the distances of the data to the line. In both cases it is easy to plot a line of best fit using a graphing calculator. The instructions are for a TI-84 calculator using the median-median method. To use the median-median method, press **STAT** and move the cursor to the right to **CALC**. Choose **3:Med-Med**. The command Med-Med will appear on the screen. Enter the two lists that contain the data **Med-Med L1, L2**. Before pressing ENTER, indicate to the calculator where to “paste” the equation in the Y= list by doing the following. Press **VARS**. Move the cursor to the right to **Y-VARS**. Choose **1:Function...** and choose Y2. The command now reads **Med-Med L1, L2, Y2**. Press ENTER. The calculator will display the equation of the line of best fit. The equation will also appear in Y2 in the Y= list. The equation (rounded to 2 places after the decimal point) for the line of best fit using the median-median method is  $y = -.87x + 72.31$ . When you press GRAPH, you will see a line connecting the two clusters.

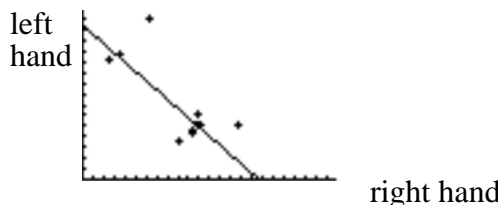


Figure 3: Med-med line of best fit for the whole group

Instructions for using the least square method are equally simple (you would choose **LinReg(ax+b)** instead of **Med-Med**). For the data in this activity, the equation for the least squares linear regression function is very similar to the one obtained above (again numbers have been rounded),  $y = -.79x + 71.9$ . The graph of the least squares line (Figure 4) looks also very similar to the previous one.

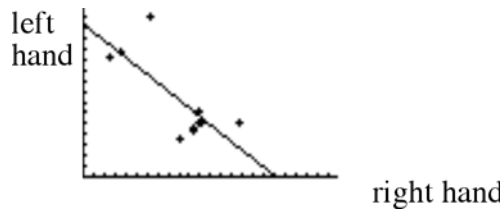


Figure 4. Least squares line of best fit

Discussion of the line of best fit for all the data:

The line of best fit for all the data has a negative slope. The interpretation would be that the more letters people write with their right hand, the fewer letters they write with their left hand. This line of fit reflects the different behavior of the two clusters. Right-handed people write more letters with their right hand than with their left hand, but for left-handed people the reverse is true. So, people who wrote very few letters with their right hand wrote many letters with their left hand. However, because we have two clearly distinct subgroups, this is a case where trying to describe the data globally can be misleading. If we look at each cluster separately, we see that within each subgroup, people who write more letters with their right hand than other people in the same group tend also to write more letters with their left hand.

### Line of best fit for each subgroup

To study the two subgroups separately, we generate two sub-lists, one containing only the data for left-handed people (Table 2) and one for right-handed people (Table 3). We then generate the line of best fit for each table (Figures 5 and 6).

$x$	number of letters right hand	18	13	32
$y$	number of letters left hand	59	56	75

Table 2: Number of letters written with each hand – left-handed people

$x$	number of letters right hand	74	52	52	55	56	54	45
$y$	number of letters left hand	26	23	22	26	26	30	18

Table 3: Number of letters written with each hand— right handed people

Line for right-handed people:  $y = .38x + 7.74$ . Line for left handed people:  $y = x + 42.33$ . Given the small numbers in each group, it is probably not very significant to focus on the difference in slope values for the graphs of left handed people and right handed people (.38 vs. 1). What is striking is that for each subgroup the slope is positive, whereas for the group as a whole the slope is negative.

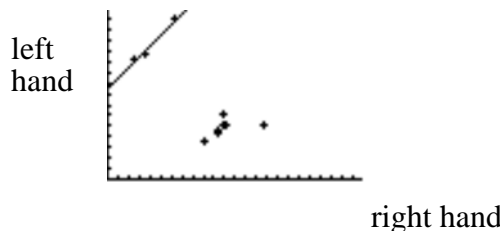


Figure 5: Best fit line for left-handed people

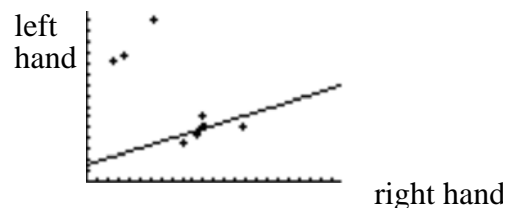


Figure 6: Best fit line for right handed people

### Dominant hand vs. non-dominant hand

A better way to think about comparing the speed of different people when writing letters would be to think in terms of dominant hand, rather than left-handed or right-handed. We can arrange the data by dominant and nondominant hand instead (Table 4), and obtain the corresponding scatter plot (Figure 7).

$x$	number letters dominant hand	74	59	52	52	56	75	55	56	54	45
$y$	number letters non-dominant hand	26	18	23	22	13	32	26	26	30	18

Table 4: Number of letters written with dominant and non-dominant hand

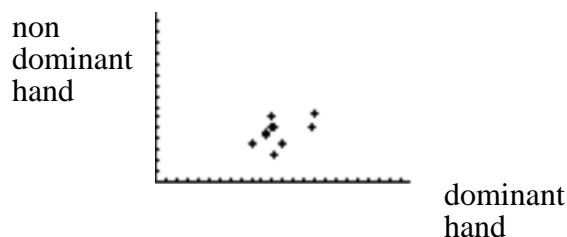


Figure 7: Letters with dominant hand vs. non-dominant hand

The equation of the med-med line of best fit (with rounded numbers) is  $y = .18x + 13.67$ . The corresponding graph is shown in Figure 8. The interpretation is that in general, people who are faster than others writing letters with their dominant hand will also tend to be faster to write letters with their non-dominant hand than others. Of course, there are individual cases in the graph where that is not the case. Can you find them? Notice also that the slope for the group as a whole is now positive (0.18). However, it is smaller than the slope of the functions when the two groups were graphed separately.

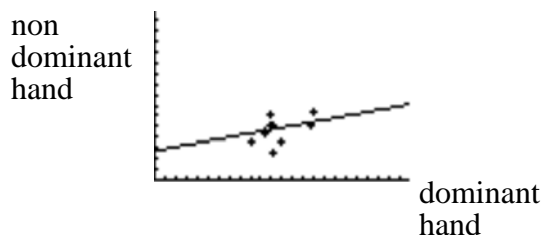


Figure 8: Best fit for dominant vs. non-dominant hand

### Concluding remarks

The example presented here is an instance where the global trend masks an opposite trend in each of the subgroups. This is not an uncommon situation. For example, an airline can have a better on-time record than another airline for each of the airports where both fly, but have an overall worse on-time record, due to the fact that the first airline flies more often to airports where there are likely to be delays due to bad weather, and the second airlines flies more often to cities where the weather is sunny most of the year. The ability to look beyond a global trend and focus on trends in subgroups is an important one. By working on examples like the one presented here, students can also learn to reconceptualize the problem and find ways to describe a situation that better reveal the relation between the variables, as they did by thinking in terms of dominant hand rather than left- or right-handedness.

### References

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